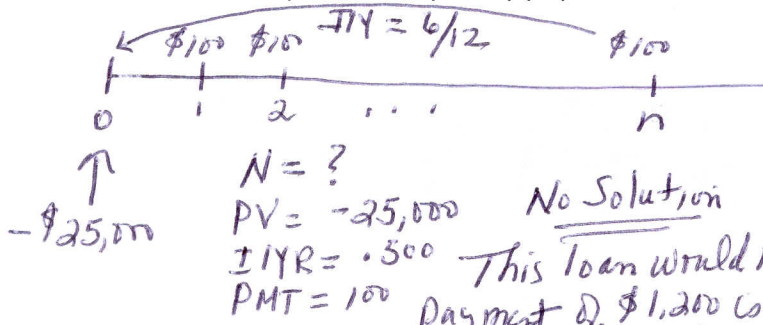


Interesting Interest Rate Problems

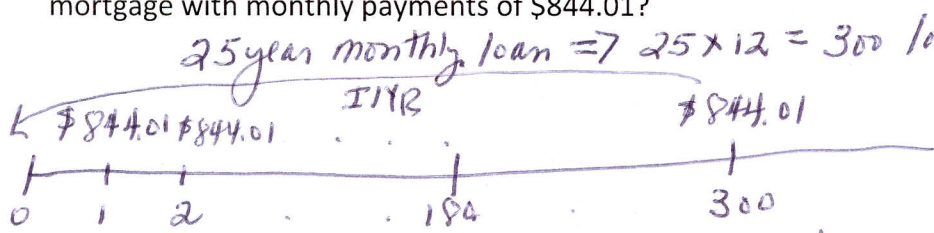
1. After completing an MBA you graduate with \$25,000 in student loans carrying 4.85% interest per annum. If you plan on paying \$100 per month on this loan, how many months will it take you to completely payoff the \$25,000 debt? Explain your answer.



- ① Convert time line to months
- ② Revise interest rate to monthly rate

No Solution
 This loan would never be paid off. The annual payment of \$1,200 is less than the interest on the loan ($6\% \times \$25,000 = \$1,500$). The debt would be increasing.

2. What would be the 15th year outstanding loan balance on a 25 year \$125,000 home mortgage with monthly payments of \$844.01?

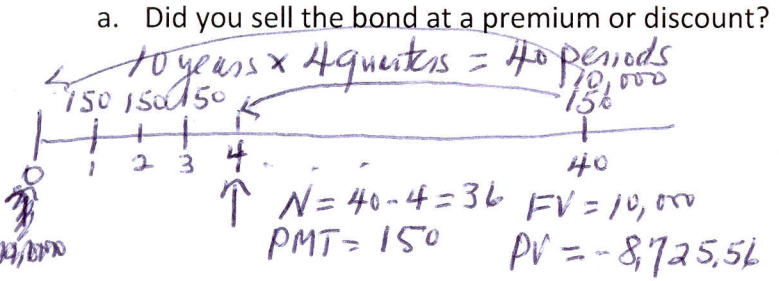


Note: The time line is in months so the I/YR rate will be the nominal monthly rate \Rightarrow to get the appropriate annual rate you must multiply the monthly rate by 12.

$N = 300$
 $PMT = 844.01$
 $PV = -125,000$
 $I/YR = .54166779$
 $\Rightarrow \text{Annual Rate} = 12 \times .54166779 = 6.5\%$

15^{th} year loan balance
 $\Rightarrow 15 \times 12 = 180^{\text{th}}$ loan balance
 $LB_{180}^{\text{pros}} = \text{prospective loan balance}$
 $\Rightarrow \text{P.V. future loan payments}$
 $N = 300 - 180 = 120$
 $PMT = 844.01$
 $I/YR = .54166779$
 $PV = \boxed{-74,330.69}$

3. The University of Iowa in response to suggestions to sell its \$150 million Jackson Pollock painting, has decided to issue general obligation bonds to fund the rebuilding of its Art Museum in the aftermath of the devastating floods of July 2010. As an alumnus you receive a call, from a local investment broker, who sells you a ten year, \$10,000 bond carrying quarterly interest of \$150. A year later when gathering money for the down payment on a home, you sell the bonds at a time when market interest rates are 8%.



Time line is in quarterly periods
 Quarterly Rate $= 8\% / 4 = 2\%$
 The bond sells at a discount, you bought at par \$10,000 sold at \$8,725.56
 Coupon Rate $\frac{150 \times 4}{10,000} = 6\% < 8\%$

b. At the time of purchase what was the stated yield rate on the bond?

When this bond was originally sold for \$10,000 the yield to maturity was exactly the same as the coupon rate of 6%. In general whenever a bond sells for the first time the coupon rate = yield to maturity.

c. Given the bond yield found in b. what was the tax equivalent yield on this security at the time of purchase?

The tax equivalent yield would be $6\% / (1 - T)$ where $T =$ your tax rate
 if $T = 28\%$, the tax equivalent yield is $6\% / (1 - .28) = 8.33\%$

4. Recently you closed on a \$250,000 fixed rate, 30 year mortgage having monthly payments of \$1,498.88. What would be the total amount of interest you would have to pay if you maintained this loan to maturity? [One Billion Dollars is not a valid answer]

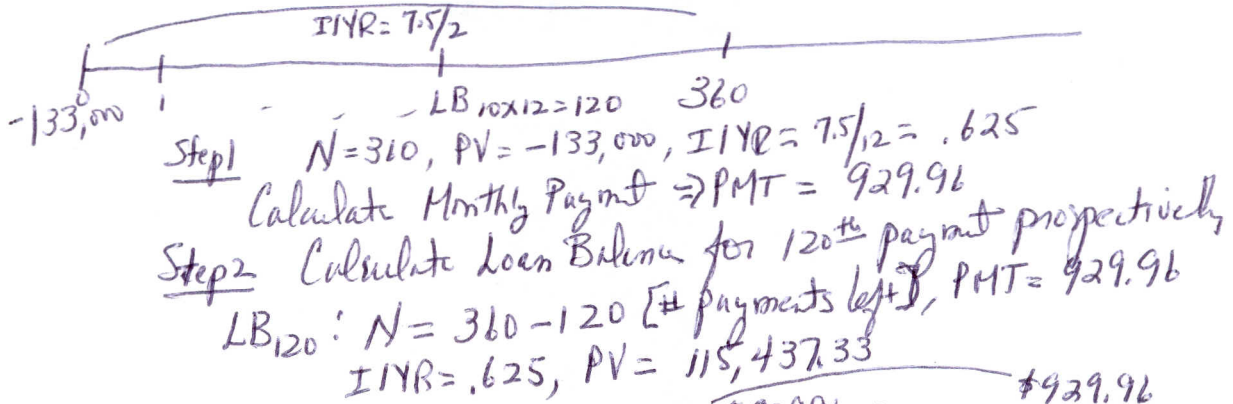
payments = $30 \times 12 = 360$

Mortgage Amortization Schedule

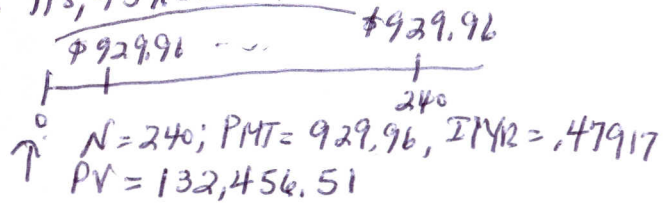
<u>Period</u>	<u>Payment</u>	<u>Interest</u>	<u>Principal Repaid</u>	<u>Outstanding Loan Balance</u>	
0				\$250,000	
1	1,498.88			Total of this Column must equal the Amt borrowed	
...					
...					
360	1,498.88				
	<u>\$539,596.80</u>		<u>\$250,000</u>		

Total Interest paid = Total of the Payments - Total Principal Repaid
 = $\$539,596.80 - \$250,000$
 = $(\$289,596.80)$

5. Ten years ago you purchased a \$167,000 home in the Cedar Valley which has recently been appraised at \$225,000. At the time of purchase you financed your home purchase with a \$133,000 thirty year, fixed rate mortgage at 7.5%. Mortgage interest rates have fallen to 5.75% and you have decided to refinance your home mortgage. You plan to borrow for 20 years, an amount that maintains your present monthly payment amount. How much extra money will you have to invest after you pay off your original 7.5% mortgage [ignore points and closing costs]?

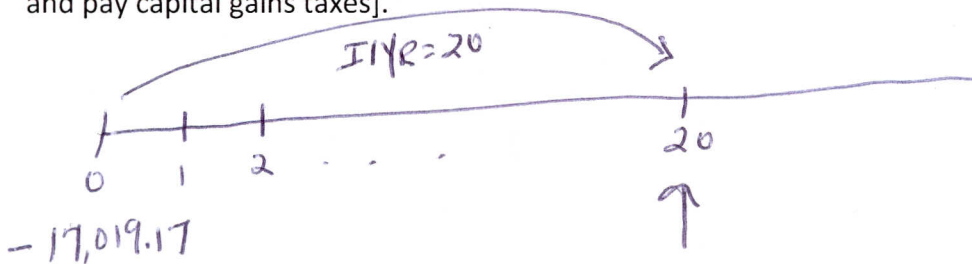


New loan: 20 years $\times 12 = 240$ payments
 Same payment, new interest rate
 $= 5.75/2 = .47917$



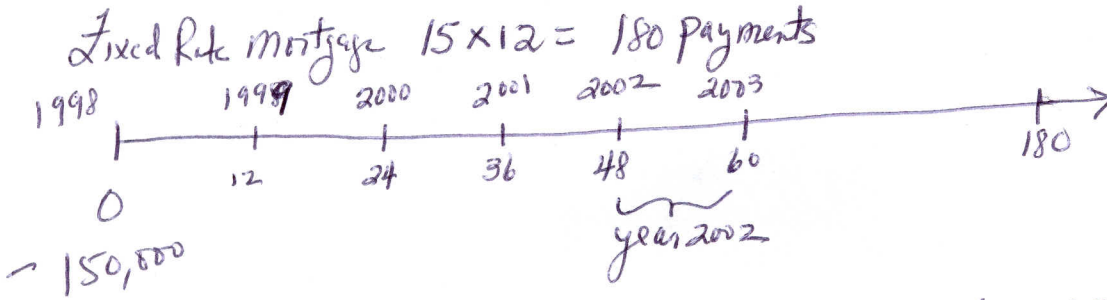
Loan Proceeds - Loan Payoff = Extra Funds. \therefore Extra Funds = $\$132,456.51 - \$115,437.33$
 $= \$17,019.17$

6. After the refinancing operation described in problem number 5 above, you invest the loan proceeds left over after paying off the original loan. You take the excess loan proceeds and invest in Berkshire Hathaway B stock generating a 20% compound annual rate of return for 20 years. What will be the value of your investment portfolio at the end of the 20 year holding period [assume that Berkshire Hathaway continues to pay no dividends, and that you are still going to retain the stock after 20 years rather than sell and pay capital gains taxes].



$N = 20$
 $PV = -17,019.17$
 $IYR = 20$
 $FV = \boxed{\$652,474.13}$

7. In 2006 you closed on a \$150,000 fixed rate, 15 year mortgage with an interest rate of 6.25%. What was the total amount of interest paid in 2010 [to be used for tax purposes, since mortgage interest is a tax deductible expense for federal income taxes]?



Step 1 Find the monthly payment: $N = 180; PV = -150,000$
 $I/YR = 6.25/12 = .520833$
 $PMT = \$1,286.13$

Step 2 Find the 48th payment loan balance prospectively
 $LB_{48}^{pros} : N = 180 - 48 = 132$ payments left
 $PMT = \$1,286.13$
 $I/YR = .520833$
 $PV = -122,547.93$

Step 3 Similarly, find the 60th payment loan balance prospectively
 $LB_{60}^{pros} : N = 180 - 60 = 120$
 $PMT = \$1,286.13$
 $I/YR = .520833$
 $PV = -114,546.95$

Difference
 = Principal
 Repaid in 2002
 \$8,000.98

Total payments in 1 year = $12 \times \$1,286.13 = \$15,433.56$
 Less: Principal Repaid $- 8,000.98$
 Interest Paid in 2002 = 7,432.58

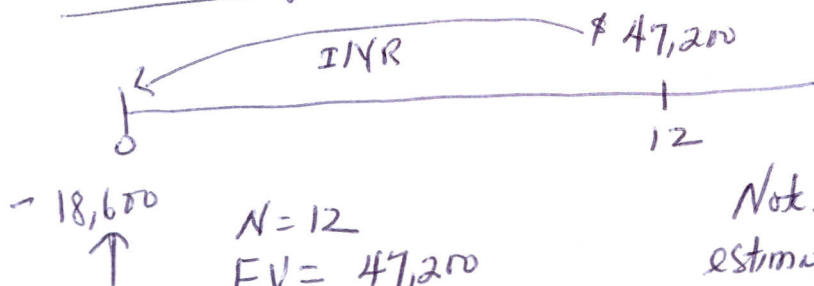
8. In 1990, after researching health care companies you decide to buy 100 shares of Merck at \$120 per share. Your total cost or original basis in this transaction was \$12,000. Six months later Merck completes a 2 for 1 stock split resulting in your account position being increased to 200 shares of Merck stock at the revised price of \$60/share [i.e., $\$120/2 = \60]. Over the following year, due to national health care initiatives in Washington DC and uncertainties about the support for drug research Merck's stock price declines from \$60 to \$33 per share. At this point, you complete a follow-up analysis of Merck and find that there are a number of promising new drugs in phase III trials any one of which could become a block buster drug. None of the financials have changed appreciably in terms of leveraging, profit margin, sales and earnings other than the external uncertainties described earlier. You purchase another 200 shares of Merck at \$33 per share increasing your original basis in the position by \$6,600 [i.e., $\$33 \times 200$], consequently, your revised basis is \$18,600 [i.e., $\$12,000 + \$6,600$]. In 1998 after several successful introductions of 5 new drug therapies, Merck's stock rises to \$90 per share at which time it is then split again 2 for 1. So, in 1998, you now have an 800 share position in Merck [i.e., $\{200 \text{ shares from original purchase} + 200 \text{ shares from later purchase}\} \times 2$] at a revised price of \$45/share [i.e., $\$90/2$]. What is the 12 year compounded rate of return on this investment assuming a revised basis of \$18,600 and that the stock is now selling for \$59 per share?

How much did you pay for your position?
 $\$18,600$

How much is that position worth?
 $800 \text{ shs} \times \$59/\text{share} = \$47,200$

What is your holding period?
 12 years

Calculate Compound Rate of Return



$$\begin{aligned}
 N &= 12 \\
 FV &= 47,200 \\
 PV &= -18,600 \\
 IYR &= 8.069 \approx 8.07\%
 \end{aligned}$$

Note: This is a Conservative estimate because you are assuming the entire purchase of \$18,600 was made in 1990 the actual compound rate of return would be slightly higher.

9. On March 15, 1995, after spending a number of fruitless hours trying to download the internet from your acreage outside Cedar Falls, you go over to Staples in search of a reliable telephone modem. The young fellow at the store points you in the direction of the modems and you ask what company is considered to be the best manufacturer in this area. He states that US Robotics is the best manufacturer and provides information on how long they've produced these modems, how many they sell, the few number that have been returned and a personal endorsement claiming he currently uses a US Robotic's modem. You buy a modem, take it home and find that it appears to be superior to every other one you've tried and returned to other stores. Once online, you decide to research the company. You download a copy of Valueline, go to Yahoo Finance, look at a couple of online annual reports and develop an analysis of the fundamental value for this stock. US Robotics is selling for \$18/ share, but has sold for as high as \$35 within the last year with a 52-week low of \$15. It has a price to earnings ratio of 18 which is remarkably low considering it is in the tech sector where P/Es tend to be quite high due to unrealistic growth expectations, cheerfully provided by knowledgeable Wall Street investment bankers. Company earnings this year were \$1.00 [e.g. $\$18/18 = \1.00]. Taking \$1.00 dividing it by \$18 gives an implied return on investment of 5.56%, while not stratospheric, is nevertheless acceptable given that interest rates are currently 4.75%. So you decide to take a flier on US Robotics and buy 200 shares at \$18 on March 31st, 1995. Two months later US Robotics reports earnings in line with expectations; however Wall Street gets excited because there was no substantial earnings growth so the stock goes to \$16 per share. You do some additional analysis to determine whether you missed something, and decide to buy 200 more shares at \$16 per share as of May 30st, 1995. In June, US Robotics makes an announcement about a new innovative product that they have developed off an earlier acquisition of a company called PALM Computing. By July when US Robotics is selling for \$35/share you decide to visit the local Staples to see what there is to be all elated about. You look at this new device called a PDA and are thoroughly unimpressed. Why would somebody pay \$150 for a hand held unit that records appointments? Nevertheless, since this investment was a purchase originally based on the fundamentals of US Robotics[i.e., earnings, leverage, modem product] without PALM[and nothing has changed in those areas] you decide to wait this out a bit more. By October US Robotics stock is selling at \$60 per share and the P/E is at a lofty multiple of 60. At this point you contemplate selling at least part of your position because your investment guidelines say you will not seek to purchase any stock with a P/E over 25 [and by the same token you would want to review with view towards selling any stock held in your portfolio that doesn't meet the criteria you set for purchase]. So, just as you get ready to sell the stock, US Robotics comes out with a wonderful communiqué stating that they will spin off 1 share of US Robotics stock into 1 share of the new PALM Company at a flotation price of \$70/share as of February 15, 1996. Outside investors may purchase additional shares of PALM at the price of \$70/share. Those holding US Robotics stock will see their shares converted directly into stock in PALM. US Robotics stock immediately rises on this announcement to \$65/share [a demonstration of short

term financial market efficiency in contrast to the long term price inefficiencies witnessed in this stock since 1995]. At this point with the P/E at 65 [i.e., \$65/\$1] and your understanding of the PALM PDA, along with the fundamentals of US Robotics without PALM included, you are committed to selling your entire position [a bird in the hand is worth many elephants in the bush]. If you sell before the end of the year you incur a short term capital gain taxed at 33% [your personal income tax rate]. If you wait until February the gain is still short term and will be taxed at the higher rate, but you might be able to create a tax loss to offset at least a portion of that gain on next year's income tax return. What to do? Fortunately, US Robotics stock rises even further based on irrational exuberance to \$69/share in December at which time you sell the 400 shares of US Robotics.

a. What was your overall after tax rate of return on this investment?

you purchased 400 shares of U.S. Robotics and sold the shares within a year \Rightarrow short term capital gain
 your basis consists of $200 \text{ shares} \times \$18 = 3,600$
 $200 \text{ shares} \times 16 = 3,200$
 $\$6,800$

you sell in Dec. 400 shares @ \$69
 \Rightarrow Net Proceeds = $400 \times \$69 = \$27,600$

Short term gain = $\$27,600 - \$6,800 = \$20,800$ Assuming you have no offsetting capital losses
 Taxes: $\$20,800 \times .33 = 6,864$
 After tax gain = $\$13,936$

Return on Investment = $\$13,936 / \$6,800 = 2.049 \approx 205\%$

b. Unfortunately, by May 31st, 1996, investors find that PALM is not capable of generating earnings at level implied by a P/E of 70 [which comes as an absolute shock to the investment community that touted the wonders of PALM], with the additional costs of the spin off and problems bringing new PALM Pilots to market, earnings drop to \$.50/share and the stock descends to earth reaching \$17/share. What is your after tax return on investment assuming at long term capital gains tax rate of 15%?

your basis is the same as in (a) \Rightarrow \$6,800
 you sell 400 shares @ \$17 = \$6,800 (Proceeds)
 your Capital gain = Proceeds - Basis
 $= \$6,800 - \$6,800 = 0$
 Return on Investment = $0 / \$6,800 = 0$

Ignoring the psychic pleasure of thinking you were rich and realizing you've been able to maintain the status quo.

c. What is the moral of this story?

A prudent investor (1) will have already defined an investment guideline or set of policies to force a partial or entire sale of this type of investment to avoid emotion getting in the way of reason (2) Selling part of an overvalued investment may mean losing participation in future gains but it is more likely to avoid incurring large losses sooner - i.e. you want to have something to tell your children